# LAKSHYA
# JEE 2024

- Subject Name - PHYSICS
- Chapter Name - Electromagnetic Induction
- Topic: Question on self inductance and mutual inductance


Lecture No.- 08

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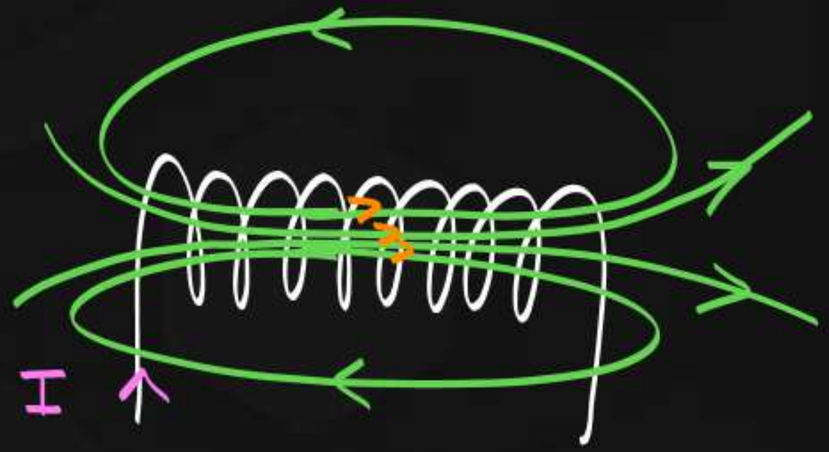
# Today's Targets

1. Self Inductance
2. Mutual Inductance
3. Questions on self and Mutual inductance

## ❖ Using magnetic energy density for magnetic energy in solenoid



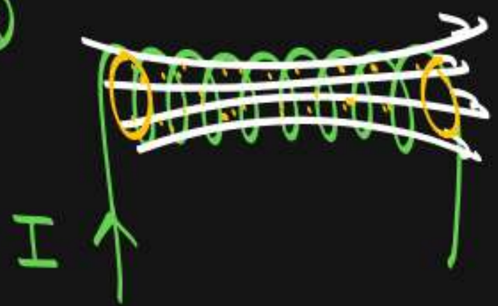

Inductor.

Energy stored in form of Magnetic field

$$\boxed{\text{Magnetic Energy density} = \frac{\text{Energy}}{\text{Volume}} = u_B = \frac{B^2}{2\mu_0}}$$

(*) Can help us to find Magnetic Energy if we know $\vec{B}$.

(*)




$B = \mu_0 n I$

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0^{\cancel{2}} n^2 I^2}{2\cancel{\mu_0}}$$

Coefficient of Self Inductance

$$\text{Total Energy} = u_B \times \text{Volume} = \frac{1}{2}(\mu_0 n^2 V) I^2 = \frac{1}{2} L I^2$$

**Question**

Find the total field energy in magnetic field stored per unit length inside a long solid cylindrical wire of radius $R$ and carrying current $I$.

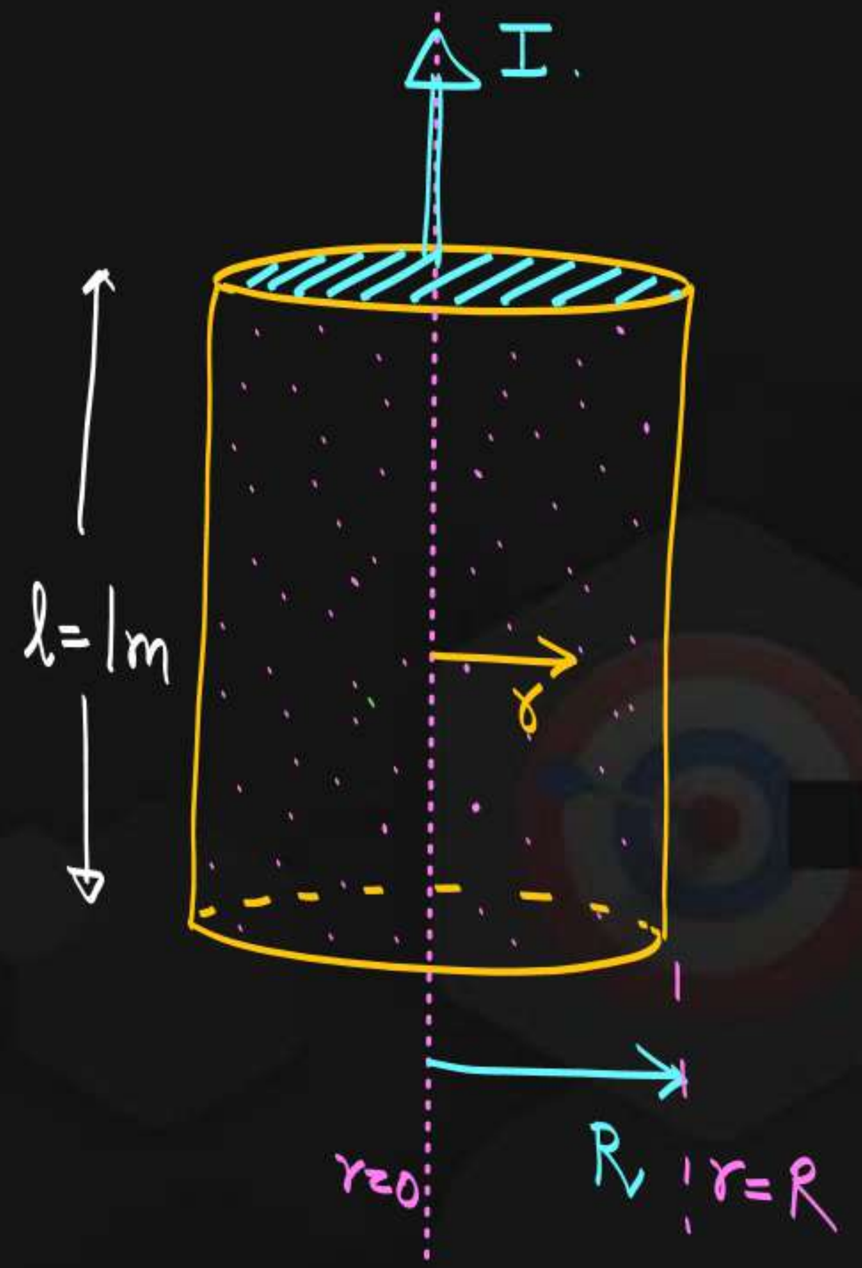
(*) $\vec{B}$ inside a Solid Cylinder

$$B_i = \frac{\mu_m I r}{2\pi R^2} \qquad u_B = \frac{B^2}{2\mu_0}$$

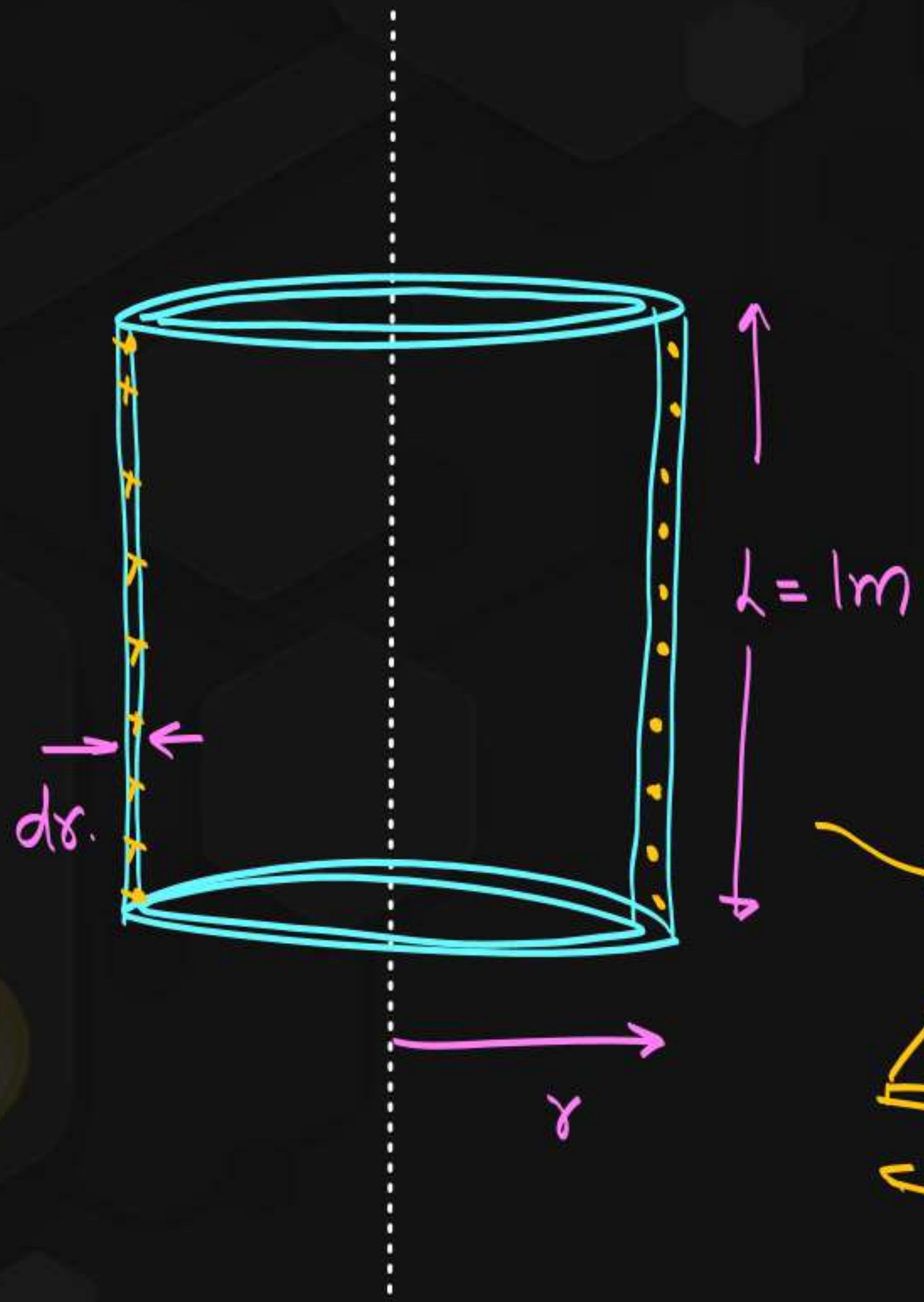
By Ampere Circuital law.

(*) $$u_B = \frac{\mu_m^2 I^2 r^2}{4\pi R^4 \cdot 2\mu_0}$$

$$\boxed{u_B = \frac{\mu_m^2 I^2}{8\pi R^4 \mu_0} r^2}$$



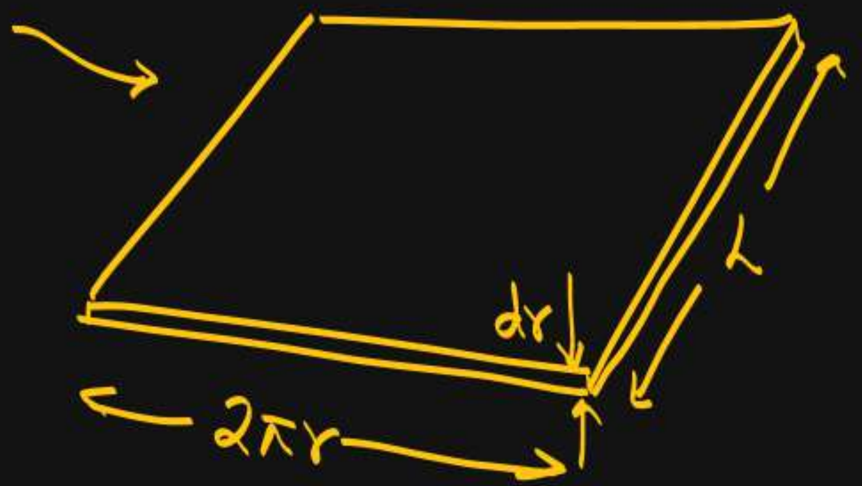
$$B = \frac{\mu_m I r}{2\pi R^2}$$




Small Magnetic Energy stored in Shell.

$$dE = u_m \cdot dV$$

$$= \frac{B^2}{2\mu_m} \cdot 2\pi r dr$$




$dV =$ Area of base $\times$ height $= (2\pi r l\, dr)$. for $l = 1m$ $dV = (2\pi r dr)$.

$$dE = \frac{\mu_m^2 I^2 r^2}{4\pi^{\cancel{2}} R^4 \times \cancel{2}\mu_0} \cdot \cancel{2\pi} r\,dr$$

$$dE = \frac{\mu_m^{\cancel{2}} I^2}{4\pi R^4 \cancel{\mu_m}} \times r^3 dr$$

$$E_T = \int_0^R \frac{\mu_m I^2}{4\pi R^4} \underbrace{r^3 dr}_{\to \text{Variable.}}$$

$$= \frac{\mu_m I^2 R^4}{16\pi R^4} = \frac{\mu_m I^2}{16\pi} \text{ // Ans.}$$

we need to Calculat Energy for unit length

$l = 1m.$

#

Summary.

$\Rightarrow$ Magnetic Energy ?

1. $u_B = \frac{\text{Energy}}{\text{Volume}} = \frac{B^2}{2\mu_0}$

$B =$ uniform: Energy $= u \times$ Volume.

$B =$ Vary: Integerate.

(*) $u_E = \dfrac{\text{Energy}}{\text{Volume}} = \dfrac{1}{2}\varepsilon_0 E^2$

1)


EF between plates of Capacitor $= \dfrac{\sigma}{\varepsilon_0}$

(uniform)

$E = \dfrac{Q}{\varepsilon_0 A}$ (uniform)

plate Area $= A$

Separation $= d$.

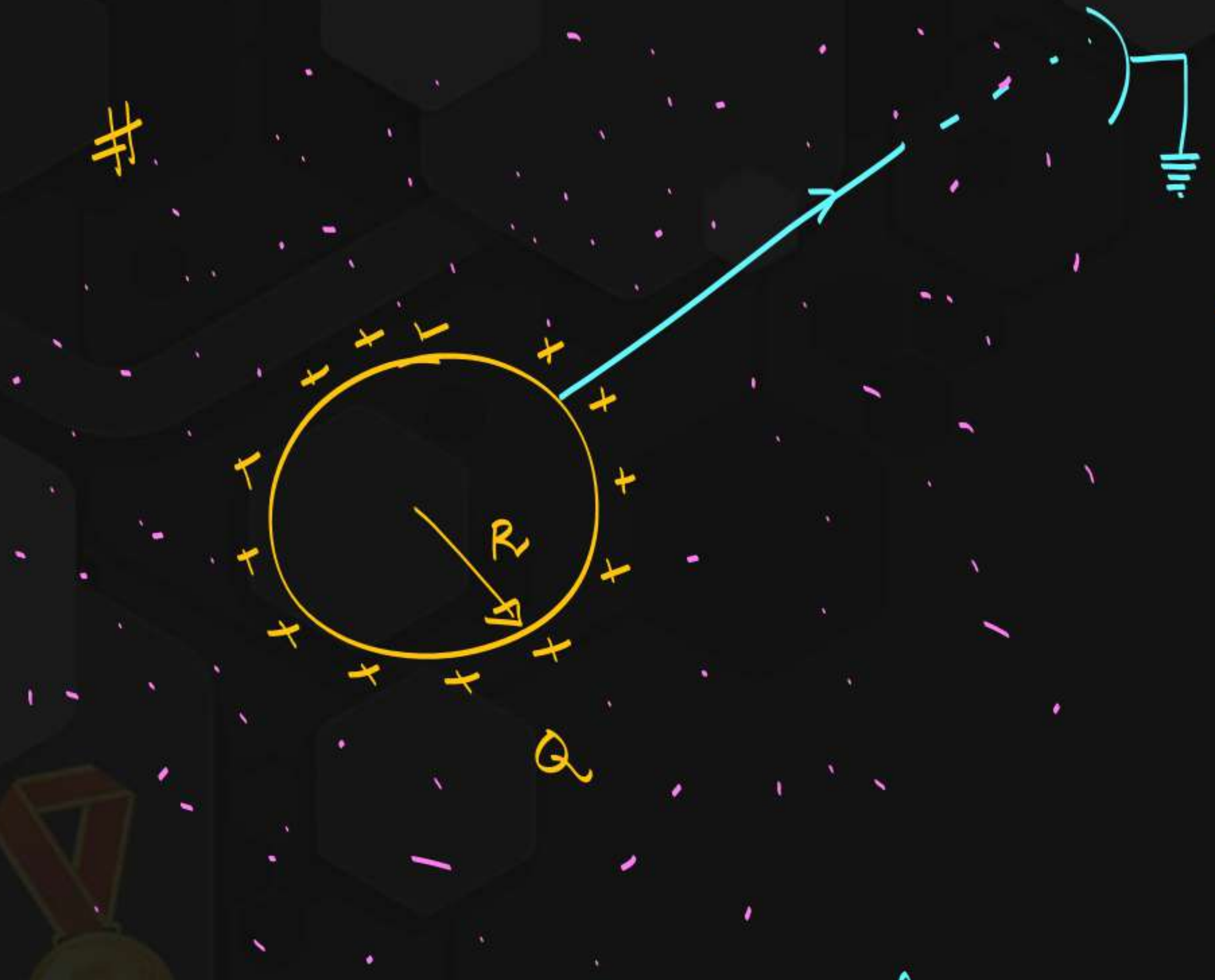
Energy between plates $= ?$

$$E_{Total} = \left(\frac{1}{2}\varepsilon_0 E^2\right) \times \text{Volume}.$$

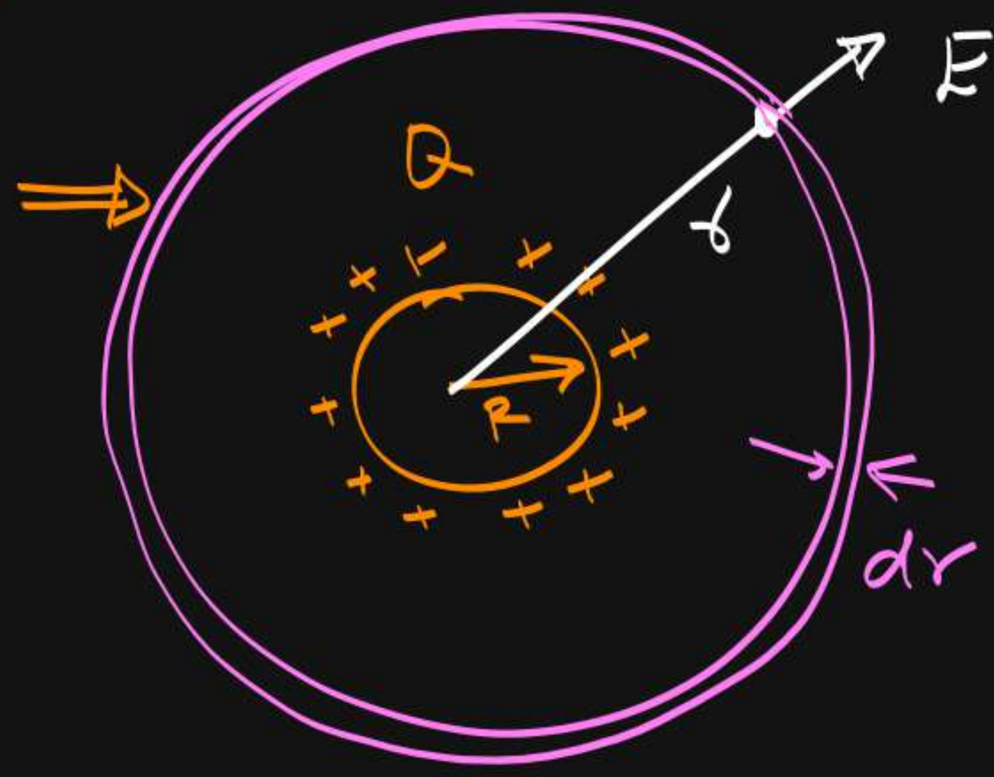
$$= \frac{1}{2}\cancel{\varepsilon_0}\,\frac{Q^2}{\varepsilon_0^{\cancel{2}} A^{\cancel{2}}}\cdot \cancel{A} d$$

$$= \frac{1}{2}\frac{Q^2}{\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{Q^2}{2C}$$

#




Find Electric Energy around a charged sphere?




$E = \frac{KQ}{r^2}$ (Vary).

$dV = 4\pi r^2 dr$.

$dE = \frac{1}{2}\varepsilon_0 \frac{K^2Q^2}{r^4} \cdot 4\pi r^2 dr$.

$dE = \frac{1}{2}\cancel{\varepsilon_0} \frac{1}{\cancel{16}_{4}\pi^2\varepsilon_0^{\cancel{2}}} \times \frac{Q^2}{r^{\cancel{4}\,2}} \cdot \cancel{4\pi r^2} dr = \frac{Q^2 dr}{8\pi\varepsilon_0 r^2}$

$$E_T = \int dE$$

$$= \int_R^{\infty} \frac{Q^2\, dr}{8\pi\varepsilon_0 r^2}$$

$$E_T = \frac{Q^2}{8\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^{\infty}$$

$$= \frac{-Q^2}{8\pi\varepsilon_0}\left[\cancel{\frac{1}{\infty}}^{\,0} - \frac{1}{R}\right]$$

$$E_T = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Q

$$E = \frac{Q^2}{2C}$$

$$= \frac{Q^2}{2(4\pi\varepsilon_0 R)}$$

$$E = \frac{Q^2}{8\pi\varepsilon_0 R}$$

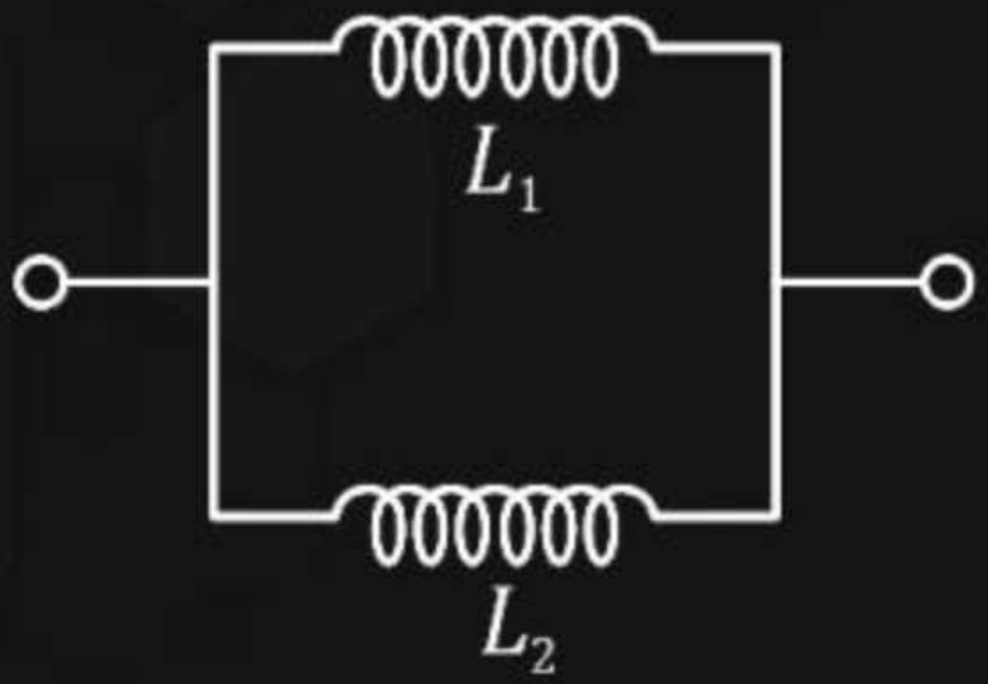
# Summary

⊛ Electric Energy

$u_E = \frac{1}{2}\varepsilon_0 E^2$

1. EF = uniform Energy = $u_E \times V$
2. EF = non uniform Energy → Integrati

# Series and Parallel of Inductance


Series Comb :- $\boxed{L_S = L_1 + L_2}$

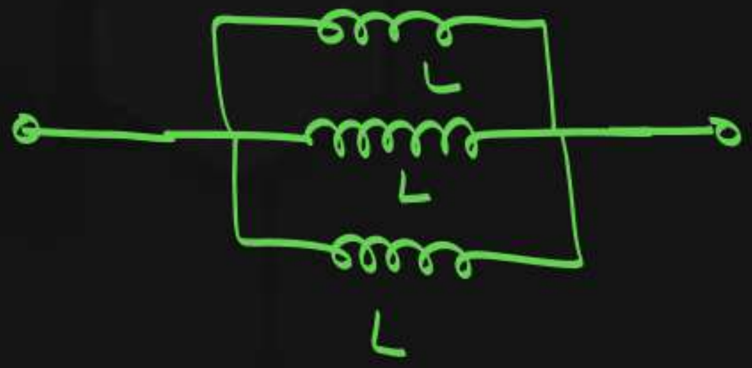
parallel Comb :- $\boxed{L_P = \frac{L_1 L_2}{L_1 + L_2}}$

Proof is not in your Syllabus.

**Question**

Find $L_{eq}$ of circuit


This is a Parallel Combination:-




$$\frac{1}{L_p} = \frac{1}{L} + \frac{1}{L} + \frac{1}{L}$$

$$\frac{1}{L_p} = \frac{3}{L}$$

$$L_p = L/3$$

Question

The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, then change in the energy of the inductance is

**(Main 2019)**

(A) 437.5 J Ans

(B) 740 J

(C) 637.5 J

(D) 540 J

$E_i = 25 = L\frac{di}{dt}$

$25 = L \times \frac{15}{1}$

$\frac{25}{15} = \boxed{L = \frac{5}{3}\text{ Henry}}$

$E = \frac{1}{2}LI^2$

$E_i = \frac{1}{2} \times \frac{5}{3} \times 10^2$

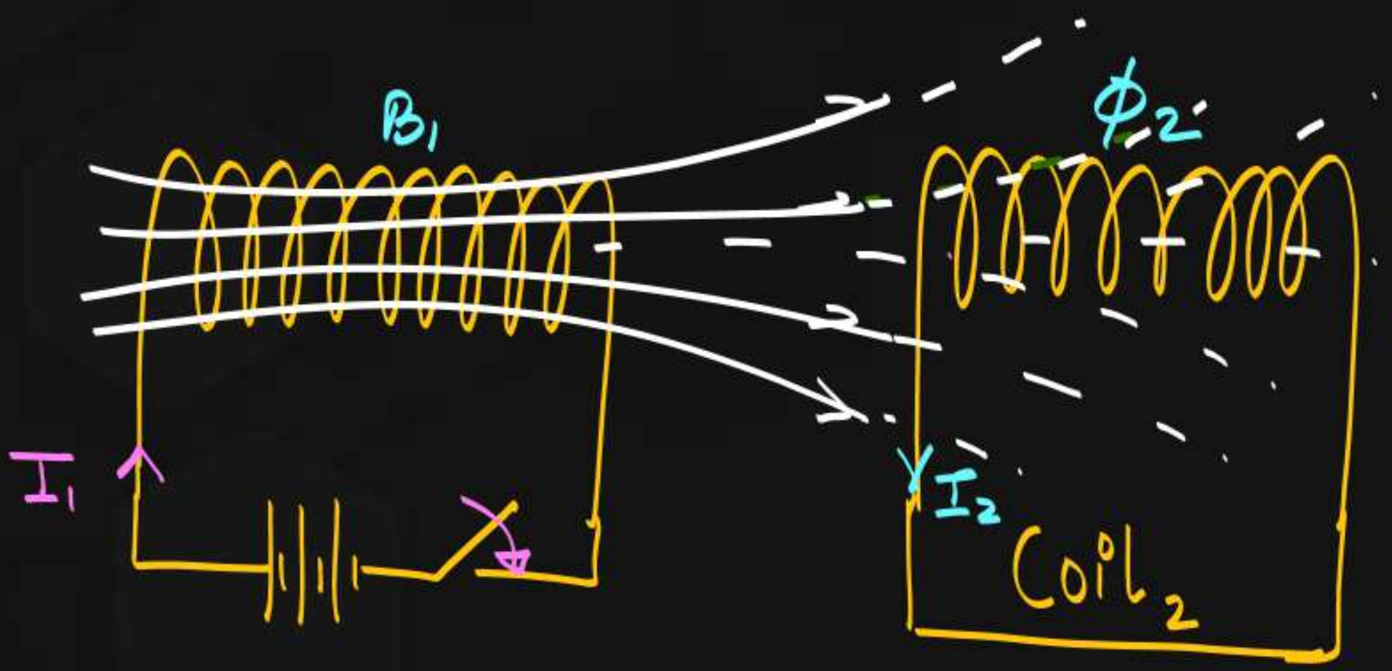
$E_f = \frac{1}{2} \times \frac{5}{3} \times 25^2$

$E_f - E_i = \frac{1}{2}\frac{5}{3} \times (25^2 - 10^2)$

$= \frac{1}{2} \times \frac{5}{\cancel{3}} \times 35 \times \cancel{15}^{5} = 437.5\text{ J}.$

# Mutual Inductance

Phenomenon by which two coil opposes change of flux in other by inducing an EMF is Called Mutual Induction




Switch close.

$I_1 \uparrow$ (slowly)

Back emf Self $= -L_1 \frac{di_1}{dt} = \varepsilon_1$

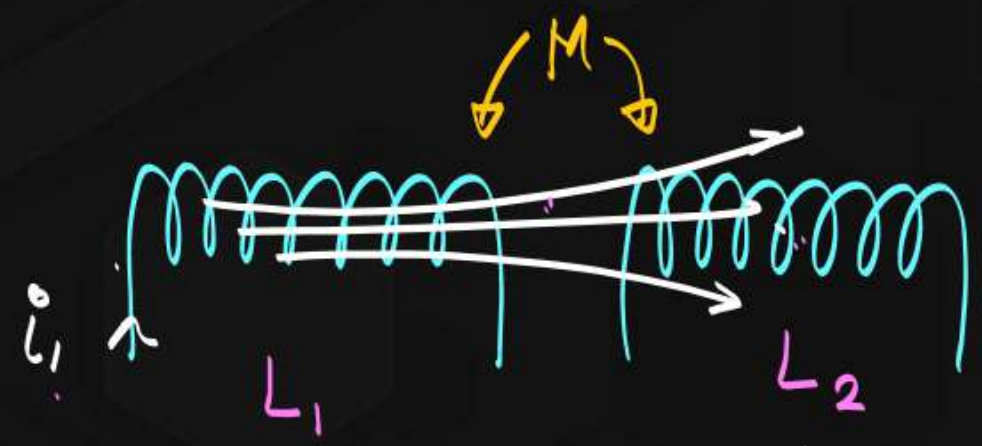
$B \uparrow$

$\phi$ in second coil $= \uparrow$

$emf_2$ to oppose change of flux in coil$_1$.

Phenomenon in which other coil also induces emf to oppose change in flux due to $I^{st}$ coil.

(*) Theory.




$\phi_2$

(*) $\phi_{\text{second coil}} \propto i_{\text{first coil}}$

$$\phi_2 = M i_1$$

M = Coefficient of Mutual Inductance unit = Henry.

If Current in Coil 1 Changes then EMF in $2^{nd}$ coil is developed.

$$\frac{d\phi_2}{dt} = M\frac{di_1}{dt}$$

$$\boxed{E_2 = -M\frac{di_1}{dt}}$$

## Question

A small coil is $r$ is placed at centre of another co-axial coil of radius $R$ ($R >> r$). Find coefficient of mutual induction ($M$)

$M = ?$

⇒ Aap kisi coil mein current bhjiye jis se hum dusri coil mein flux likh paayi.

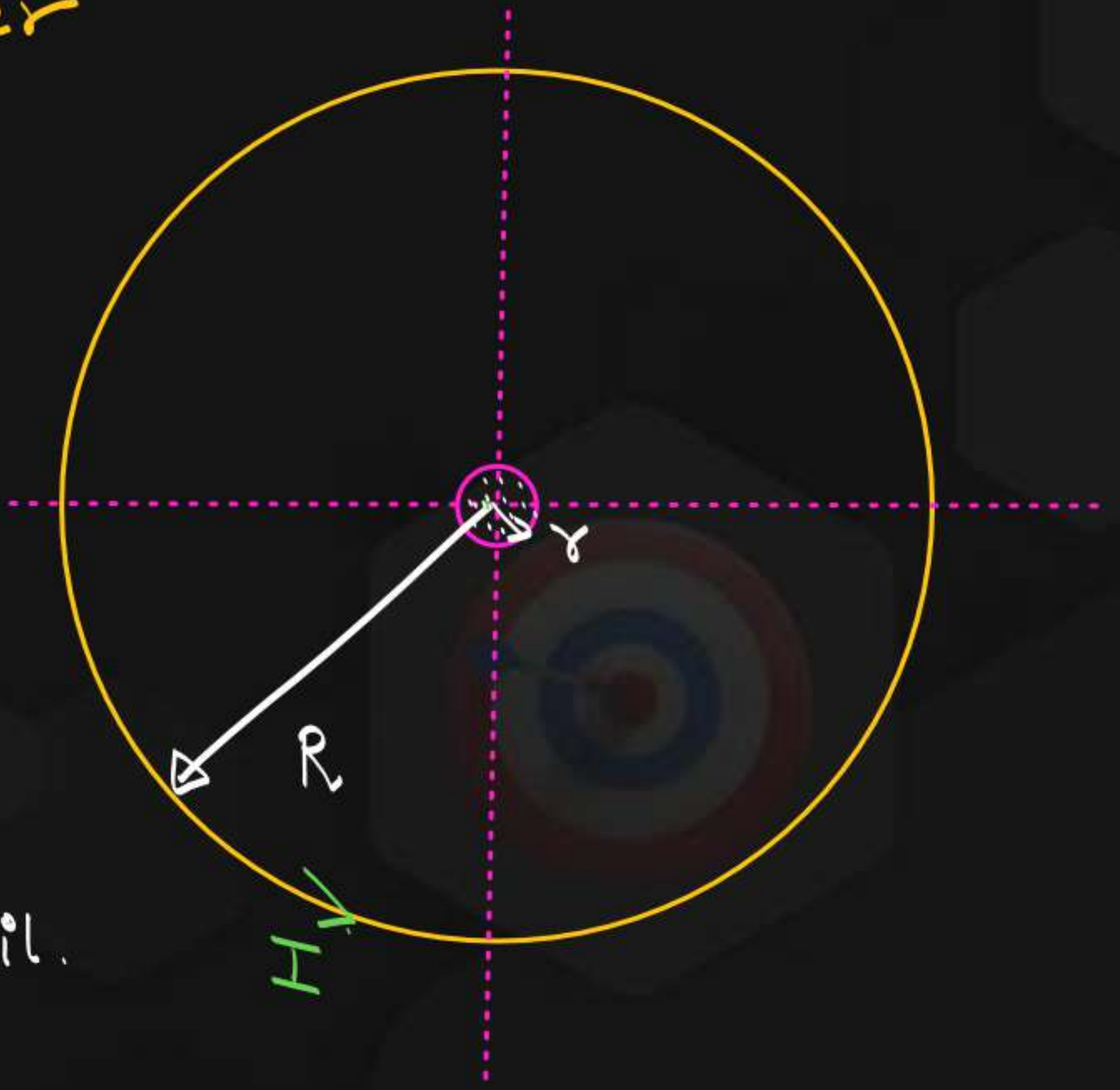
→ $\phi_{\text{second coil}} = M\, i_{\text{first}}$

(*) Let current in bigger coil $= I$.

$$B_{\text{centre}} = \frac{\mu_0 I}{2R}$$

$$\phi_{\text{second coil}} = \left(\frac{\mu_0 I}{2R}\right) \cdot \pi r^2$$

$$\phi_{\text{second coil}} = \underbrace{\frac{\mu_0 \pi r^2}{2R}}_{M} I_{\text{first coil}}$$



**Question**

A small square frame is placed at centre of bigger square frame of side $(L \times L)$. If smaller square $(a \times a)$ has current $I$. Find coefficient of mutual inductance.

only for Confusion.

L, M doesnot depend on which loop has Current.


$$B_{cente} = 4\,B_{wire}$$

$$= 4\left[\frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)} \cdot 2\sin 45\right]$$

$$B_c = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}$$

$$\phi_{smaller} = \frac{2\sqrt{2}\,\mu_0 I}{\pi L} \cdot a^2$$

$$\phi_{smaller} = \boxed{\frac{2\sqrt{2}\mu_0 a^2}{\pi L}} I_{bigger}$$

Coefficient of Mutual Inductance.

# Homework

- All DPP of EMI 01,02,03,04,05

- Module Question of Emi: All questions before mutual inductance

Target → Electric flux & Earthing of Conductors

Notes → Revise, (Notes → Quest)

OR

⊛ Moving charge & Magnetism

Notes → Revise, (Notes → Ques).

Sunday PYQ's


THANK
YOU